

SYSTEM AND METHOD FOR PROVIDING A SCALABLE
OBJECTIVE METRIC FOR AUTOMATIC VIDEO QUALITY EVALUATION
EMPLOYING INTERDEPENDENT OBJECTIVE METRICS

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REFERENCE TO PROVISIONAL APPLICATION

This patent application refers to and claims the priority and benefit of Provisional Patent Application Serial No. 60/260,842 filed January 10, 2001.

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RELATED APPLICATIONS

This patent application is related to co-pending United States Patent Application Serial No. 09/734,823 filed December 12, 2000 by Ali et al. entitled "System and Method for Providing a Scalable Dynamic Objective Metric for Automatic Video Quality Evaluation." The present invention is related to that disclosed in United States Patent Application Serial No. 09/817,981 filed March 27, 2001 by Ali et al. entitled "System and Method for Optimizing Control Parameter Settings in a Chain of Video Processing Algorithms."

Both of the related patent applications are commonly assigned to the assignee of the present invention. The disclosures of both of the related patent applications are hereby incorporated by reference in the present application as if fully set forth herein.

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TECHNICAL FIELD OF THE INVENTION

The present invention is generally directed to systems and methods for evaluating video quality, and, in particular, to an improved system and method for providing a scalable objective metric employing interdependent objective metrics for automatically evaluating video quality of a video image.

BACKGROUND OF THE INVENTION

Video experts continually seek new algorithms and methods for improving the quality of video images. The primary goal is to obtain the most perceptually appealing video image possible. The ultimate criterion is the question "How well does the viewer like the resulting picture?" One way to answer the question is to have a panel of viewers watch certain video sequences and then record the opinions of the viewers concerning the resulting image quality. The results, however, will vary from panel to panel according to the variability between the viewing panels. This problem is commonly encountered when relying on subjective human opinion. The severity of the problem is increased when the viewing panel is composed of non-experts.

Results solely based upon on human perception and subjective

opinion are usually subjected to subsequent statistical analysis to remove ambiguities that result from the non-deterministic nature of subjective results. Linear and non-linear heuristic statistical models have been proposed to normalize these types of subjective results and obtain certain figures of merit that represent the goodness (or the degradation) of video quality. The process of measuring video quality in this manner is referred to as "subjective video quality assessment."

Subjective video quality assessment methods give valid indications of visible video artifacts. Subjective video quality assessment methods, however, are probabilistic in nature, complex, time consuming, and sometimes difficult to apply. In addition, there is a problem in selecting appropriate viewers for the viewing panel. A non-trained viewer will be a poor judge of the suitability of new video processing methods. A non-trained viewer, however, will likely accurately represent the general consumers in the marketplace. On the other hand, a trained expert viewer will be overly biased toward detecting minor defects that will never be noticed by the general consumer.

To avoid the disadvantages that attend subjective methods for evaluating video quality, it is desirable to use automated

objective methods to evaluate video quality. Automated objective methods seek to obtain objective figure of merits to quantify the goodness (or the degradation) of video quality. The process for obtaining one or more objective measures of the video quality must
5 be automated in order to be able to quickly analyze differing types of video algorithms as the video algorithms sequentially appear in a video stream.

Objective measures of video quality are fully deterministic. That is, the results will always be the same when the test is
10 repeated (assuming the same settings are preserved).

Because the ultimate goal is to present the viewer with the most appealing picture, a final judge of the value of the objective measures of video quality is the degree of correlation that the objective measures have with the subjective results. Statistical
15 analysis is usually used to correlate the results objectively obtained (automatically generated) with the results subjectively obtained (from human opinion).

There is a need in the art for improved systems and methods for automatically measuring video quality. The process of
20 automatically measuring video quality is referred to as "objective video quality assessment."

Several different types of algorithms have been proposed that are capable of providing objective video quality assessment. The algorithms are generally referred to as "objective video quality models." A report from the Video Quality Experts Group (VQEG) sets forth and describes the results of an evaluation performed on ten (10) objective video quality models. The report is dated December 1999 and is entitled "Final Report from the Video Quality Experts Group on the validation of Objective Models of Video Quality Assessment." The report is presently available on the World Wide Web at <http://www-ext.crc.ca/VQEG>.

Each different objective video quality model provides its own distinctive measurement of video quality referred to as an "objective metric." A "double ended" objective metric is one that evaluates video quality using a first original video image and a second processed video image. A "double ended" objective metric compares the first original video image to the second processed video image to evaluate video quality by determining changes in the original video image. A "single ended" objective metric is one that evaluates video quality without referring to the original video image. A "single ended" objective metric applies an algorithm to a video image to evaluate its quality.

No single objective metric has been found to be superior to all the other objective metrics under all conditions and for all video artifacts. Each objective metric has its own advantages and disadvantages. Objective metrics differ widely in performance (i.e., how well their results correlate with subjective quality assessment results), and in stability (i.e., how well they handle different types of video artifacts), and in complexity (i.e., how much computation power is needed to perform the algorithm calculations).

A wide range of applications exists to which objective metrics may be applied. For example, fast real-time objective metrics are needed to judge the quality of a broadcast video signal. On the other hand, more complex and reliable objective metrics are better for judging the quality of non-real time video simulations.

Using only one objective metric (and one objective video quality model) limits the evaluation of the quality of a video signal to the level of evaluation that is obtainable from the objective metric that is used. It is therefore desirable to use more than one objective metric for video quality evaluation. An improved system and method that uses more than one objective metric for video quality evaluation has been disclosed in United

[illegible]

5 There is a need in the art for an improved system and method
for combining objective metrics in order to form more efficient
objective metrics for video quality evaluation.

SUMMARY OF THE INVENTION

The present invention generally comprises an improved system and method for providing a scalable objective metric employing
5 interdependent objective metrics for automatically evaluating video quality of a video image.

In an advantageous embodiment of the present invention, the improved system of the invention comprises an objective metric controller that is capable of receiving a plurality of objective
10 metric figures of merit from a plurality of objective metric model units. The objective metric controller is capable of using objective metrics for both desirable and undesirable video image characteristics. The objective metric controller is also capable of using a plurality of interdependent objective metrics. The
15 objective metric controller is capable of determining a scalable objective metric from the plurality of interdependent objective figures of merit.

In an advantageous embodiment of the present invention, the improved method of the invention comprises the steps of
20 (1) receiving in an objective metric controller a plurality of objective metric figures of merit from a plurality of objective

metric model units comprising at least one pair of objective metric model units that is interdependent, and (2) determining a scalable objective metric from the plurality of the objective metric figures of merit.

5 It is a primary object of the present invention to provide an improved system and method for providing a scalable objective metric for automatically evaluating video quality of a video image using interdependent object metric model units.

10 It is another object of the present invention to provide a scalable objective metric from a correlation factor derived from a mathematical description of an interdependency of at least one interdependent pair of objective metric model units.

15 It is an additional object of the present invention to provide a scalable objective metric from correlation factor derived using a neural network algorithm that employs both objective quality scores and subjective quality scores.

20 It is another object of the present invention to continually determine new values of the scalable objective metric from new values of the plurality of objective metric figures of merit as new video images are continually received.

The foregoing has outlined rather broadly the features and

technical advantages of the present invention so that those skilled in the art may better understand the Detailed Description of the Invention that follows. Additional features and advantages of the invention will be described hereinafter that form the subject of the claims of the invention. Those skilled in the art should appreciate that they may readily use the conception and the specific embodiment disclosed as a basis for modifying or designing other structures for carrying out the same purposes of the present invention. Those skilled in the art should also realize that such equivalent constructions do not depart from the spirit and scope of the invention in its broadest form.

Before undertaking the Detailed Description of the Invention, it may be advantageous to set forth definitions of certain words and phrases used throughout this patent document: the terms "include" and "comprise" and derivatives thereof, mean inclusion without limitation; the term "or," is inclusive, meaning and/or; the phrases "associated with" and "associated therewith," as well as derivatives thereof, may mean to include, be included within, interconnect with, contain, be contained within, connect to or with, couple to or with, be communicable with, cooperate with, interleave, juxtapose, be proximate to, be bound to or with, have,

have a property of, or the like; and the term "controller,"
"processor," or "apparatus" means any device, system or part
thereof that controls at least one operation, such a device may be
implemented in hardware, firmware or software, or some combination
5 of at least two of the same. It should be noted that the
functionality associated with any particular controller may be
centralized or distributed, whether locally or remotely.

Definitions for certain words and phrases are provided throughout
this patent document, those of ordinary skill in the art should
10 understand that in many, if not most instances, such definitions
apply to prior, as well as future uses of such defined words and
phrases.

BRIEF DESCRIPTION OF THE DRAWINGS

For a more complete understanding of the present invention, and the advantages thereof, reference is now made to the following
5 descriptions taken in conjunction with the accompanying drawings, wherein like numbers designate like objects, and in which:

FIGURE 1 is a block diagram that illustrates (1) a plurality
of objective metric model units for obtaining a plurality of
objective metric figures of merit from a video stream and
10 (2) a objective metric controller capable of using the plurality of
objective metric figures of merit to determine a scalable objective
metric;

FIGURE 2 is a flow chart diagram illustrating an advantageous
method of using a plurality of objective metric figures of merit to
15 determine a scalable objective metric;

FIGURE 3 is a flow chart diagram illustrating an advantageous
method of operation of the improved system and method of the
present invention; and

FIGURE 4 is a flow chart diagram illustrating an alternative
20 advantageous method of operation of the improved system and method
of the present invention.

DETAILED DESCRIPTION OF THE INVENTION

FIGURES 1 through 4, discussed below, and the various embodiments set forth in this patent document to describe the principles of the improved system and method of the present invention are by way of illustration only and should not be construed in any way to limit the scope of the invention. Those skilled in the art will readily understand that the principles of the present invention may also be successfully applied in any type of device for evaluating video quality.

FIGURE 1 illustrates system 100 for providing a scalable objective metric for automatic video quality evaluation. System 100 receives video stream 110. Each of a plurality of objective metric model units (120, 130, ... , 140) receives a copy of the video signal of video stream 110. Objective metric model unit 120 applies a first objective metric model (referred to as "Metric 1") to obtain a first figure of merit, $f(1)$, that represents the quality of the video signal based on the first objective metric model. The first figure of merit, $f(1)$, is provided to controller 150.

Similarly, objective metric model unit 130 applies a second objective metric model (referred to as "Metric 2") to obtain a

second figure of merit, $f(2)$, that represents the quality of the video signal based on the second objective metric model. The second figure of merit, $f(2)$, is also provided to controller 150. Continuing in this manner, other objective metric model units are added until the last objective metric model unit 140 has been added. Objective metric model unit 140 applies the last objective metric model (referred to as "Metric N"). Objective metric model units (120, 130, ..., 140) obtain a plurality of figures of merit ($f(1)$, $f(2)$, ..., $f(N)$) and provide them to controller 150.

The figures of merit ($f(1)$, $f(2)$, ..., $f(N)$) represent a series of N evaluations of the quality of the video stream by N different objective metrics. The figures of merit ($f(1)$, $f(2)$, ..., $f(N)$) may also be designated $f(i)$ where the value of i goes from 1 to N.

As will be explained below in greater detail, system 100 of the present invention provides a system and method for using the figures of merit $f(i)$ to calculate a scalable objective metric. The letter "F" (shown in FIGURE 1) designates the scalable objective metric of the present invention.

System 100 of the present invention comprises controller 150 and memory 160. Controller 150 may comprise a conventional

microprocessor chip or specially designed hardware. Controller 150 is coupled to a plurality of objective metric model units (120, 130, ... , 140) via signal communication lines (shown in FIGURE 1). Controller 150 operates in conjunction with an operating system (not shown) located within memory 160 to process data, to store data, to retrieve data and to output data. Controller 150 calculates scalable objective metric "F" by executing computer instructions stored in memory 160.

Memory 160 may comprise random access memory (RAM), read only memory (ROM), or a combination of random access memory (RAM) and read only memory (ROM). In an advantageous embodiment of the present invention, memory 160 may comprise a non-volatile random access memory (RAM), such as flash memory. Memory 160 may also comprise a mass storage data device, such as a hard disk drive (not shown in FIGURE 1) or a compact disk read only memory (CD-ROM) (not shown in FIGURE 1).

It is noted that the system and method of the present invention may be used in a wide variety of types of video processing systems, including, without limitation, hard disk drive based television sets and hard disk drive based video recorders, such as a ReplayTV™ video recorder or a TiVO™ video recorder.

Controller 150 and metric calculation algorithm 170 together comprise an objective metric controller that is capable of carrying out the present invention. Under the direction of computer instructions in metric calculation algorithm 170 stored within memory 160, controller 150 calculates a scalable objective metric "F" using the figures of merit $f(i)$.

A weighting unit 190 within controller 150 dynamically detects the currently occurring characteristics of the video sequence. The currently occurring characteristics may include such features as sharpness, color, saturation, motion, and similar types of features. Weighting unit 190 assigns a value (or "weight") $w(i)$ to each objective metric (Metric 1, Metric 2, ..., Metric N). For example, if Metric 1 is especially good when used on a certain first type of video signal, then the value of $w(1)$ is given a greater value than the other values of $w(i)$. Conversely, if Metric 2 is not very good when used on that same first type of video signal, then $w(2)$ will be given a lower value than the other values of $w(i)$. If a second type of video signal is present, it may be that Metric 1 is not as good as Metric 2 when used on the second type of video signal. In that case, $w(2)$ is given a higher value and $w(1)$ is given a lower value than the other values of $w(i)$.

Generally speaking, the values of $w(i)$ that weighting unit 190 selects will vary depending upon the type of video signal that weighing unit 190 dynamically detects. Controller 150 uses metric calculation algorithm 170 to calculate the sum S of the products of
 5 each $w(i)$ and $f(i)$. That is,

$$S = w(1)f(1) + w(2)f(2) + \dots + w(N)f(N) \quad (1)$$

or
$$S = \sum w(i)f(i) \quad (2)$$

where the value of i runs from 1 to N .

A correlation factor $r(i)$ is associated with each figure of merit $f(i)$. The correlation factor $r(i)$ is obtained from the expression:

$$r(i) = 1 - [A(i) / B] \quad (3)$$

where

$$A(i) = 6 \sum [(X(i,j) - Y(i,j))]^2 \quad (4)$$

where the value of j runs from 1 to n .

and where

$$B = n (n^2 - 1) \quad (5)$$

The values of $X(i,j)$ are the values of a set of n objective data values for a video image. The values of $Y(i,j)$ are the values of a set of n subjective data values for the same video image. That is, the number of X data points (n) is the same number of Y data points
5 (n).

The value $r(i)$ is referred to as the "Spearman rank" correlation factor. The value $r(i)$ is a measure of how well the objective X values match the subjective Y values. The values of the correlations factors $r(i)$ for each figure of merit $f(i)$ are
10 known, having been previously determined by statistical analysis. Values of the correlation factors $r(i)$ are stored in metric parameters look up table 180 in memory 150.

A "best fitting" value for scalable objective metric "F" is desired. The "best fitting" value of "F" represents the highest
15 level of correlation of the objective metric measurements of video quality (generated automatically) and the subjective measurements of video quality (from human opinions). The "best fitting" value of "F" represents the closest approximation of the subjective measurements of video quality by the objective measurements of
20 video quality. Because the video images in a video stream are constantly changing, the "best fit" will require constant automatic

updating. The term "dynamic" refers to the ability of the objective metric of the present invention to continually change its value to take into account the continual changes of the video images in a video stream.

5 As previously mentioned, weighting unit 190 continually (i.e., dynamically) detects the characteristics of the video sequence as they occur. For each correlation factor $r(i)$, weighting unit 190 continually assigns values of $w(i)$ to each figure of merit $f(i)$. To dynamically obtain the "best fitting" value of "F", metric calculation algorithm 170 determines the values of $w(i)$ that cause the value S to be a maximum for each value of $r(i)$. The largest of these values (i.e., the maximum value) is selected to be the scalable objective metric "F." That is,

$$F = \text{Maximum} [S(r(1)), S(r(2)), \dots, S(r(N))] \quad (6)$$

Scalable objective metric "F" is referred to as "scalable" because new objective metric model units can be easily added (as long as their correlation factors $r(i)$ are defined). In addition, objective metric model units that are no longer desired can easily be removed.

The scalable objective metric "F" of the present invention provides a great deal of flexibility. For example, for fast (real time) video signals, any complicated measurement objective metrics may be switched off so that their figures of merit are not considered in the metric calculation process. For simulation and video chain optimization applications, where more time can be used to perform the metric calculation, the more complicated measurement objective metrics may be switched on so that their figures of merit may be considered in the metric calculation process.

The scalable objective metric of the present invention avoids the shortcomings of any single objective metric. This is because weighting unit 190 will assign a low value to $w(i)$ for any objective metric that performs poorly in the presence of a certain set of artifacts. The scalable objective metric of the present invention achieves the highest correlation with the results of subjective testing when compared any single objective metric. The scalable objective metric of the present invention will be at least as good as the best single objective metric under all circumstances. Because the scalable objective metric permits the inclusion of any objective metric, the system and method of the present invention is not limited to use with a particular type of

objective metric (e.g., a "single ended" objective metric or a "double ended" objective metric).

It is noted that the elements of the present invention that have been implemented in software (e.g., weighting unit 190) may be
5 implemented in hardware if so desired.

As shown in FIGURE 1, system 100 of the present invention also comprises neural network unit 195. In one embodiment neural network unit 195 may be located within controller 150. The operation of neural network unit 195 will be more fully described
10 below.

FIGURE 2 is a flow chart diagram illustrating the method of operation of the system of the present invention. The steps of the method are generally denoted with reference numeral 200. A video image from video stream 110 is provided to N objective metric model
15 units (120, 130, ... , 140). The N objective metric model units (120, 130, ... , 140) evaluate the video image and obtain N respective figures of merit, $f(i)$ (step 205).

Weighting unit 190 in objective metric controller 150 then dynamically detects video characteristics of the video image and
20 assigns N weights, $w(i)$, to the N figures of merit, $f(i)$ (step 210). For each correlation factor, $r(i)$, objective metric

controller 150 calculates a sum, $S(r(i))$, that is equal to the sum of each product of weight, $w(i)$, and figure of merit, $f(i)$ (step 215).

Objective metric controller 150 then selects the maximum value of the sum, $S(r(i))$, that corresponds to the best correlation of objective measurements of video quality with subjective measurements of video quality (step 220). Objective metric controller 150 then assigns that value to be the value of the scalable objective metric "F" (step 225). Objective metric controller 150 then outputs that value of "F" (step 230).

After the value of "F" has been output, a determination is made whether objective metric controller 150 is still receiving video images (decision step 235). If the video has ended, then the process ends. If the video has not ended and more video images are being received, control passes back to step 205 and the objective controller 150 continues to operate in the manner that has been described.

The present invention has been described as a system for providing a scalable objective metric for evaluating video quality of a video image. It is understood that the "scalable objective metric" of the present invention is a general case that includes as

a subset the more specific case of providing "static objective metric." To provide a "static objective metric" the present invention 1) receives a plurality of objective metric figures of merit from a plurality of objective metric model units, and 2) 5 determines a weight value, $w(i)$, for each of the plurality of objective metric figures of merit, and 3) thereafter keeps the weight values, $w(i)$, constant (i.e., unchanged) during the process of calculating objective metric "F" for video stream 110.

The present invention also comprises a system and method for calculating an objective metric "F" by using both single objective metrics that represent desired image features and single objective metrics that represent undesired image features. Examples of desired image features are sharpness and contrast. Examples of undesired image features are noise, blockiness, and aliasing. 10
15 A dynamic objective metric "F" that produces good results may be obtained by using competing single objective metrics. That is, single objective metrics that representing both desired and undesired image features are to be combined.

The single objective metrics may be interdependent. 20 For example, consider a simple sharpness objective metric that is dependent on the presence of noise in the image. Let the sharpness

of an image be represented by the signal power P_H in a high frequency band B_H . Enhancing the sharpness of the image will increase the signal power in this frequency band to P_H' where P_H' is equal to P_H plus the change in P_H (i.e., ΔP_H). This may be
 5 expressed as:

$$P_H' = P_H + \Delta P_H \quad (7)$$

The measured signal power is an indication of the image sharpness.

Adding white noise to the clean image will also increase the signal power to:

$$P_H'' = P_H + N_H \quad (8)$$

Where N_H is the noise power in frequency band B_H . The sharpness metric should therefore be defined as the total signal power P_H minus the measured noise power N_H . The sharpness metric is interdependent on the noise metric.

15 If single objective metrics are used that are not interdependent, then scalable objective metric "F" is calculated as in Equation (6). The weight factors that are assigned to desired features are given the opposite sign of weight factors that are assigned to undesired features.

20 If single objective metrics are used that are interdependent, then scalable objective metric "F" is not

necessarily a linear function of the values of the single objective metrics. When interdependent single objective metrics are present the value of the scalable objective metric "F" may be determined by (1) describing the interdependencies with mathematical equations, and (2) correlating the images that correspond to the interdependencies with subjective quality scores.

Alternatively, the value of the scalable objective metric "F" may be determined by using a neural network algorithm that employs both objective quality scores and subjective quality scores. In this embodiment of the invention, controller 150 employs neural network unit 195 to calculate a value of the scalable objective metric "F" from the values of the interdependent objective metrics. In one embodiment, neural network unit 195 is located within controller 150. In other embodiments, neural network unit 195 may be located externally to controller 150.

FIGURE 3 is a flow chart diagram illustrating an alternate method of operation of the system of the present invention. The steps of the method are generally denoted with reference numeral 300. A video image from video stream 110 is provided to N objective metric model units (120, 130, ... , 140). The N objective metric model units (120, 130, ... , 140) evaluate the video image and

obtain N respective figures of merit, $f(i)$ (step 305).

Weighting unit 190 in objective metric controller 150 then dynamically detects video characteristics of the video image and assigns N weights, $w(i)$, to the N figures of merit, $f(i)$ (step 310). For independent (i.e., non-interdependent) objective metrics, objective metric controller 150 calculates a sum, $S(r(i))$, using a correlation factor, $r(i)$. The sum, $S(r(i))$, is equal to the sum of each product of weight, $w(i)$, and figure of merit, $f(i)$ (step 315).

For independent (i.e., non-interdependent) objective metrics, objective metric controller 150 then selects the maximum value of the sum, $S(r(i))$, that corresponds to the best correlation of objective measurements of video quality with subjective measurements of video quality (step 320). Objective metric controller 150 then assigns that value to be the value of the scalable objective metric "F" (step 325).

For interdependent objective metrics, objective metric controller 150 calculates a value of the scalable objective metric "F" from a mathematical description of the interdependencies of the interdependent objective metrics (Step 330).

Objective metric controller 150 then outputs the value of "F"

(step 335). After the value of "F" has been output, a determination is made whether objective metric controller 150 is still receiving video images (decision step 340). If the video has ended, then the process ends. If the video has not ended and more video images are being received, control passes back to step 305 and the objective controller 150 continues to operate in the manner that has been described.

FIGURE 4 is a flow chart diagram illustrating an alternate method of operation of the system of the present invention. The steps of the method are generally denoted with reference numeral 400. A video image from video stream 110 is provided to N objective metric model units (120, 130, ... , 140). The N objective metric model units (120, 130, ... , 140) evaluate the video image and obtain N respective figures of merit, $f(i)$ (step 410).

For independent or interdependent objective metrics, objective metric controller 150 uses neural network unit 195 to calculate a value of the scalable objective metric "F" from the values of the interdependent objective metrics (Step 420). The neural network algorithm in neural network unit 195 has previously been trained with subjective video quality scores.

Objective metric controller 150 then outputs the value of "F"

(step 430). After the value of "F" has been output, a determination is made whether objective metric controller 150 is still receiving video images (decision step 440). If the video has ended, then the process ends. If the video has not ended and more video images are
5 being received, control passes back to step 410 and the objective controller 150 continues to operate in the manner that has been described.

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